Elasticitetsmodel

A calibrated surrogate of LTM 2.3 for cost and time changes, with effects by income and car ownership

Model documentation, DTU Management

October 2026

Abstract

Ministries often need the effect of a fuel tax, a kilometre charge or a public-transport fare change within days, and they also need to know which households are affected. The Danish national model LTM 2.3 answers the first question, but each run takes considerable time, and its results are not easily split by income and car ownership. In this note, we document a fast surrogate model that reproduces the LTM 2.3 elasticities by trip purpose and mode, applies them to the 2023 travel pattern from the Danish National Travel Survey, and reports the results by income quintile, household car ownership and tour length. The parameters are not estimated from observed choices. They are calibrated so that the model reproduces the elasticities of LTM 2.3, which are themselves the output of an estimated disaggregate model. The default model is a nested logit over the joint choice of mode and tour length, which has the nesting structure of the LTM mode and destination model. Destinations are represented by tour length, because a surrogate without zones cannot represent destinations directly. The logsum parameter is calibrated rather than taken from LTM, because a length class aggregates many zones and therefore substitutes differently from a zone. A multinomial logit and an inverse product differentiation logit over modes are documented as alternatives. The nested logit reproduces the LTM tour elasticities with a root mean square error between 0.017 and 0.029 per purpose and the mileage elasticities with an error between 0.017 and 0.035. A 10% increase in the car cost per kilometer reduces car vehicle-kilometers by 3.5% and raises public-transport person-kilometers by 2.5% in 2023.

1 Introduction

Large transport models such as LTM 2.3 (Rich, 2022; Rich and Hansen, 2016) and its successor GMM 4.0 (GMM 4.0 project, 2026) are the standard instruments for assessing pricing policies in Denmark. In this note, we document a surrogate model that reproduces the LTM 2.3 elasticities for four uniform changes in level of service, namely the car cost per kilometer, the car travel time, the public-transport fare and the public-transport travel time, and applies them to the 2023 travel pattern. The model is intended for quick policy questions, where a full model run is too slow and where the distribution of the effects across households is part of the question. It is not a network model. A change in a cost or a time applies to every origin and destination at once, and there is no route, zone or time-of-day detail. For this reason, the model complements LTM and GMM rather than replacing them.

The model differs from a standard demand model in four respects, and the note is written to make these differences transparent. Firstly, the parameters are not estimated from observed choices but calibrated so that the model reproduces the elasticities of LTM 2.3 as closely as possible. The elasticities of the model are therefore model output, and Section 4.2 and Appendix B compare every one of them with its LTM value. Secondly, the default demand structure is a nested logit over the joint choice of mode and tour length. It takes the nesting structure of the LTM mode and destination model (Rich, 2022, Table 5.1), but it represents a destination by its tour length, because the model has no zones. Thirdly, cost sharing between drivers and

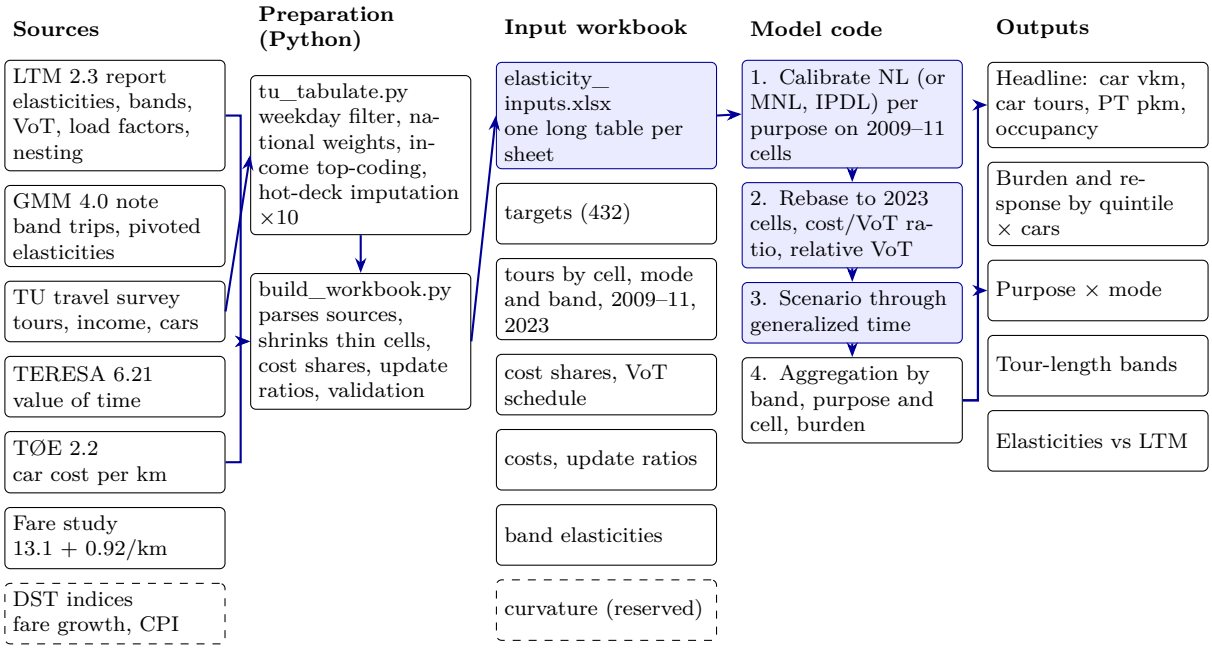

Figure 1: Data flow from the sources (left) through preparation, the input workbook and the model code to the outputs (right). Dashed boxes are inputs that are still placeholders or reserved for later model runs.

passengers, as specified by Rich and Fox (2024), is built into the money component of generalized time, so that a change in car cost reaches drivers with passengers and car passengers at the correct strength. Fourthly, the calibration is carried out on the income and car-ownership cells to which the model is applied, which removes an aggregation bias that would otherwise reduce the segmented elasticities.

Two alternatives to the nested logit are documented and available in the explorer. The multinomial logit over the same alternatives is the special case with a logsum parameter of one, and it is the easiest model to explain. The inverse product differentiation logit (IPDL) of Fosgerau et al. (2022) over modes was the first version of the model. It has overlapping nests between modes, which LTM does not have, and it handles tour length outside the choice model. Section 3.3 describes both, and Section 4.2 compares the three models by their fit and by cross-validation.

Figure 1 shows how data flow through the model. The sources on the left are prepared by two Python programs and collected in one input workbook. The model code reads the workbook, calibrates the parameters on the 2009–11 cells, rebases them to 2023, and computes the outputs for a given scenario. The same code runs in a web page and in Node, so that the published explorer and the test suite use identical calculations.

The note is organized as follows. Section 2 describes the data. Section 3 presents the nested logit over mode and tour length, generalized time, cost sharing and the two alternative models. Section 4 describes the calibration, compares the calibrated elasticities with those of LTM and describes the rebasing to 2023. Section 5 defines the outputs and reports the main results. Section 6 discusses the relation to standard models and the limitations. Table 3 at the start of Section 3 collects the notation, Appendix A lists the input workbook, and Appendix B compares the LTM and model elasticities for every purpose.

2 Data

Travel pattern. The travel pattern comes from the Danish National Travel Survey (TU, DTU Management, 2025b). We use closed home-based journeys, i.e. tours that start and end at home, made on weekdays outside June to August by persons aged 10 to 84. This definition follows the weekday concept of LTM and GMM. Each tour is assigned a purpose from its primary activity (commute, education, escort, shopping, leisure, business) and a main mode (walk, bike, car driver alone, car driver with passengers, car passenger, public transport). Car tours are split into the three car modes from the longest car leg of the tour, where a driver with at least one companion counts as a driver with passengers. Each tour is also assigned to one of six tour-length bands (0–20, 20–50, 50–100, 100–200, 200–300 and above 300 km, out and home). The survey weight scales each survey year to the national population for an average day. After the weekday filter, the weights are rescaled within each year so that the remaining sessions again sum to the population of that year, and they are divided by the number of pooled years. Hence, every count is a national number of tours per average weekday. The 2023 base pools 2018–19 and 2022–24 and excludes the covid years, while the calibration period pools 2009–11. The base contains 6.37 million tours per weekday, and Table 1 shows the distribution by purpose and mode.

Table 1: Base 2023: tours per average weekday by purpose and main mode (thousands), and mean tour length (km, out and home). Source: TU, 2018–19 and 2022–24, weighted to national level.

	Walk	Bike	SOV	HOV	Passenger	PT	All
Commute	82	354	1 100	62	65	179	1 842
Education	104	225	51	13	110	161	664
Escort	60	41	131	169	8	4	413
Shopping	307	120	458	187	135	42	1 250
Leisure	783	237	413	204	244	87	1 969
Business	4	14	176	14	10	13	232
All	1 342	991	2 328	649	573	487	6 370
Mean tour km	2.6	7.9	46.0	34.9	32.7	49.2	28.9

Income and car ownership. Household income is deflated to 2023 prices and top-coded at the weighted 99.5th percentile. Between 30% and 47% of the sessions lack income, depending on the period and the interview mode. We impute the missing incomes by weighted random hot-deck. A missing household and personal income is replaced by the incomes of a donor drawn with probability proportional to the survey weight from the same imputation class. The classes are defined by age, occupation, household cars, number of adults, education, region and urbanity, and they are collapsed step by step until a class contains at least five donors. The imputation is repeated ten times, and every table reports the mean over the imputations. Persons are then grouped into weighted quintiles of household income, and the cells of the model are the combinations of quintile and household cars (none, one, two or more). The imputed and observed income distributions agree closely, with medians of 693 and 706 thousand DKK in the 2023 base.

Table 2 shows the number of persons, the mean income and the household car ownership by quintile for both periods. In 2023, the share of persons in households without a car falls from 41% in the lowest quintile to 4% in the highest. The last column is the mean number of household cars seen from a person, with two or more cars counted as two. It is not the number of cars per person, because two adults in a two-car household both count two cars. The measure rises from 0.67 in the lowest quintile to 1.59 in the highest. Measured as the ratio of the log changes between the lowest and the highest quintile, its cross-sectional income elasticity is 0.46 in 2023 and 0.53 in 2009–11. The two values indicate a modest decline in the income dependence of car ownership over the period, and they are background for the experimental car-ownership

extension (Section 5.1).

Table 2: Persons aged 10–84 (thousands), mean household and personal income (1000 DKK, 2023 prices) and household car ownership (% of persons) by income quintile. Household cars is the person-weighted mean number of household cars, with two or more counted as two. The elasticity row is the ratio of the log changes in household cars and household income between the lowest and the highest quintile. Source: TU, 2009–11 and 2018–19, 2022–24, weighted to national level, incomes imputed by hot-deck.

		Persons	Mean income		Household cars (% of persons)			Household
Period	Quintile	(1000)	Household	Personal	None	One	Two+	cars
2009–11	Q1	961	218	157	45.7	49.8	4.6	0.59
	Q2	962	438	266	21.9	64.1	14.0	0.92
	Q3	942	673	299	7.9	57.5	34.5	1.27
	Q4	976	874	362	5.0	51.4	43.6	1.39
	Q5	976	1 333	514	2.7	42.0	55.2	1.53
Income elasticity of household cars, Q1 to Q5								0.53
2023	Q1	1 028	227	173	40.5	52.0	7.5	0.67
	Q2	1 032	460	285	18.0	63.5	18.6	1.01
	Q3	1 032	700	335	8.3	48.7	43.0	1.35
	Q4	1 037	930	375	4.7	39.2	56.1	1.51
	Q5	1 034	1 469	549	4.4	32.7	63.0	1.59
Income elasticity of household cars, Q1 to Q5								0.46

Thin cells. With six purposes, five quintiles, three car classes, six modes and six bands, many cells have few observations. We therefore shrink the observed shares of a cell toward the shares of its purpose and car class,

$$\tilde{w}_{ca} = \frac{n_c \, \hat{w}_{ca} + \tau \, \bar{w}_{pka}}{n_c + \tau}, \tag{1}$$

where a is a mode for the IPDL and a combination of mode and band for the nested logit, $\hat{w}_{ca}$ is the observed share in cell c, $\bar{w}_{pka}$ is the share in purpose p and car class k, n_c is the unweighted number of tours in the cell, and $\tau = 30$. Eq. (1) has the form of the posterior mean of the shares under a Dirichlet prior with mean $\bar{w}_{pk\cdot}$ and concentration τ, so τ acts as a number of pseudo-observations. The analogy is not exact, because the observed shares are survey weighted and n_c is a mean over the imputations. A cell with $n_c = 300$ tours keeps 91% of its own shares, while a cell with $n_c = 10$ keeps 25%. Shrinkage toward the car class keeps some driving in cells without a household car, which we interpret as the use of borrowed, rented and company cars.

Elasticity targets. The targets are the elasticities of the primary-tour model of LTM 2.3 (Rich, 2022, Tables 6.28–6.45). For each purpose and mode, the report gives arc elasticities of tours, of average tour length and of mileage for a 10% increase in the car cost, the car time, the public-transport fare and the public-transport time. The same report gives the tour elasticities in the six tour-length bands for the car and public-transport modes, the value of time by income band (Table 5.7), the load factors used for cost sharing (Table 5.8), and the nesting structure and logsum parameters of the mode and destination model (Tables 5.1 and 8.25). The nested logit is calibrated to the tour, length and mileage elasticities and to the band elasticities, while the IPDL is calibrated to the tour elasticities only. We apply the primary-tour elasticities to all tours. The pivoted elasticities of Pedersen (2021) are kept as a comparison and are not targets.

Costs and prices. The car cost follows the LTM definition of fuel and tyres. We take it from the marginal cost per kilometer including taxes in TØE 2.2 (DTU Management, 2025a), which gives 1.162 DKK/km in 2010 and 1.088 DKK/km in 2023 at 2025 prices. The total operating cost in TERESA (DTU Management, 2021) is not used, because its repair component contains a method break between 2020 and 2021 that reduces the total by about one fifth. The value of time grows by a factor 1.197 between 2010 and 2023 according to TERESA, which follows

the growth in income. Rich and Vandet (2019) analyse how the value of travel time savings in Denmark has developed over time, including the years of the financial crisis. The public-transport fare is the linear fit of Argyros et al. (2026) to the Danish zone fares, 13.1 DKK plus 0.92 DKK per kilometer for a one-way trip, which gives $2 \cdot 13.1 + 0.92\,\ell$ for a tour of length ℓ.

3 Model

In this section, we present the nested logit over mode and tour length, the representation of destinations by tour length, the generalized time through which the scenarios enter, and the cost sharing between car occupants. The two alternative models are described at the end of the section. Table 3 collects the notation used in this and the following sections. The cell index c and the purpose index p are omitted where they are not needed.

3.1 Choice of mode and tour length

Nesting in LTM. The mode and destination model of LTM 2.3 is a nested logit, and its nesting structure differs between purposes (Rich, 2022, Tables 5.1 and 8.49). For commuting and business, the destination is chosen in the upper level and the mode in the lower level, $p(m|d)\,p(d)$. We call this structure B. For education, escort, shopping and leisure, the mode is chosen in the upper level and the destination in the lower level, $p(d|m)\,p(m)$, which we call structure A. In both structures, the utility of the lower level enters the upper level as a logsum multiplied by the logsum parameter θ. The estimated values are 0.918 for commuting, 0.504 for education, 0.506 for escort, 0.567 for shopping, 0.560 for leisure and 0.708 for business (Rich, 2022, Table 8.25). A value below one means that the alternatives of the upper level are less close substitutes than the alternatives of the lower level. For commuting with $\theta = 0.918$, a commuter faced with a higher car cost changes mode more readily than workplace, and the difference between the two is small. There is no nesting among the modes themselves in LTM. The model of this note takes over this structure. It is a nested logit over the joint choice of mode and destination, with the two possible orderings of the levels, and it has no nests between modes.

Destinations represented by tour length. The surrogate has no zones, so a destination cannot be represented directly. However, the elasticities that the model has to reproduce do not depend on the identity of a destination. They depend on how the change in utility varies across destinations, and for a uniform change in cost or time this variation comes from the length of the tour. We therefore aggregate all destinations at the same tour length into one alternative and represent a destination by its tour length. Each of the six bands is divided into equal intervals with midpoints ℓ_{bu}, namely 40 intervals for the band below 20 km, 12 for each of the four middle bands, and 16 for the band above 300 km, which is closed at $300 + 8(\bar{\ell}_{cjb} - 300)$ km. A node (b, u) thus plays the role of a destination, and an alternative $a = (j, b, u)$ is a mode at a tour length. All modes share the same nodes, so in structure B a destination node contains every mode, as a zone does in LTM. The base tours of an alternative are

$$T^0_{ca} = \tilde{N}_{cjb}\,\omega_{cjbu}, \qquad \sum_u \omega_{cjbu} = 1, \qquad \sum_u \omega_{cjbu}\,\ell_{bu} = \bar{\ell}_{cjb}, \tag{2}$$

where $\tilde{N}_{cjb} = N_c \tilde{w}_{c(j,b)}$ are the shrunk tours of mode j in band b, and ω_{cjbu} distributes them over the nodes so that the mean length of the band is the observed one. For the motorized modes, ω_{cjbu} is a truncated exponential density over the band, $\omega_{cjbu} \propto \exp(-r_{cjb}(\ell_{bu} - \ell^{\mathrm{lo}}_b))$, with r_{cjb} chosen to match the mean. The rate can be negative, so the density can also increase within a band.

Walk and bike. Walk and bike have their own low speeds, and their share falls much faster with distance than that of the other modes. LTM represents this by utilities that are linear in walk and bike time with large coefficients, which are -0.156 and -0.091 per minute of walking

Table 3: Notation. Indices are listed first, then data, parameters, model quantities and outputs.

Symbol	Meaning
Indices and sets	
p	Trip purpose: commute, education, escort, shopping, leisure, business
$j \in \mathcal{J}$	Mode: walk (W), bike (B), car driver alone (S), car driver with passengers (H), car passenger (C), public transport (P)
$\mathcal{A} = \{\mathrm{S}, \mathrm{H}, \mathrm{C}, \mathrm{P}\}$	Modes with a level-of-service shock
b, u	Tour-length band ($b = 1, \ldots, 6$) and length node within the band
$a = (j, b, u)$	Alternative of the nested logit: a mode at a tour length
$n(a)$	Nest of alternative a: its length node (b, u) in structure B, its mode j in structure A
0	The no-tour option
q, k	Income quintile ($q = 1, \ldots, 5$) and household cars ($k \in \{0, 1, 2+\}$)
$c = (q, k)$	Cell; $\mathcal{C}^{10}$ and $\mathcal{C}^{23}$ are the cells of 2009–11 and 2023
κ	Shock: car cost (CC), car time (CT), public-transport fare (PC), public-transport time (PT)
Data	
N_c, n_c	Weighted and unweighted number of tours in cell c
N_{cjb}, $\bar{\ell}_{cjb}$	Tours and mean tour length (km, out and home) of mode j in band b
ℓ_a, ω_a	Length of node a and its share of the tours of mode j in band b
P_c	Persons aged 10–84 in cell c
$E_{j\kappa}$, $E^{\ell}_{j\kappa}$, $E^{\mathrm{km}}_{j\kappa}$	LTM 2.3 arc elasticity of tours, average tour length and mileage at +10%
$e_{jb\kappa}$	LTM 2.3 tour elasticity in band b
L_{H}, L_{C}	Adult load factors of a car with passengers and of a passenger
y, $\bar{y}$, ρ_q	Personal income, average personal income, relative income of quintile q
$v(y)$	LTM value of time at personal income y
γ	Fuel and tyre cost per car-kilometer
$f(\ell)$	Public-transport fare of a tour of length ℓ
Parameters	
θ	Logsum parameter of the nested logit, $0 < \theta \leq 1$
β_j	Strength of the response of mode $j \in \mathcal{A}$ to generalized time
φ^{T}, φ^{C}	Length exponents of the time and cost response, for car and for public transport
κ_j	LTM walk and bike time disutility per tour-kilometer
σ_{S}, σ_{P}	Money share of generalized time for SOV and public transport at calibration
α_j	Share tilt at calibration ($\alpha_{\mathrm{S}} = 0$)
s_0	No-tour share (fixed at 0.01)
τ	Prior strength of the shrinkage (30 pseudo-observations)
π_m	Prior money shares
r_{car}, r_{PT}	Growth of cost relative to value of time, 2010–2023
μ_d	Nesting weight of dimension d (IPDL only)
Model quantities	
V_a, Δ_a	Utility of alternative a and its change in a scenario
W_n	Change in the inclusive value of nest n, in exponential form
T_j, C_j, G_j	Travel time, money cost and generalized time
s^{c}_j	Share of the car cost carried by the traveler
σ_j, o_j	Money share of generalized time and the money-to-time odds $o_j = \sigma_j/(1-\sigma_j)$
x_κ	Proportional shock
$\hat{E}_{j\kappa}$	Model arc elasticity
Outputs	
T^0_{ca}, T^1_{ca}, K^0_{cj}, K^1_{cj}	Tours and kilometers before and after the shock
B_c	Money burden of cell c per weekday

for commuting inside and outside the Copenhagen area, and -0.131 and -0.134 per minute of cycling (Rich, 2022, Table 8.25). We use these coefficients to shape the length distribution of walk and bike within a band,

$$\omega_{cjbu} \propto \ell_{bu}^{\eta_{cjb}}\, e^{-\kappa_j \ell_{bu}}, \qquad \kappa_j = -\frac{60}{2\, v_j}\, \bar{\beta}_j^{\mathrm{LTM}}, \qquad j \in \{\mathrm{W}, \mathrm{B}\}, \tag{3}$$

where $\bar{\beta}_j^{\mathrm{LTM}}$ is the mean of the two LTM time coefficients of the purpose, v_j is the speed (5 km/h walking and 15 km/h cycling), and the factor 2 converts the one-way time of LTM to the out-and-home length of a tour. For commuting, $\kappa_{\mathrm{W}} = 0.742$ and $\kappa_{\mathrm{B}} = 0.265$ per tour-kilometer. The exponent η_{cjb} is chosen so that the mean length matches the observed one, and it describes how the number of possible destinations grows with distance. For commuting in the band below 20 km, the mean exponent over the cells is 0.93. A value of one is what one would expect, because the area of a ring at distance ℓ around the home, and hence the number of destinations in it, grows in proportion to ℓ. The agreement between the LTM time coefficients, the assumed speeds and the observed mean lengths of walking and cycling therefore supports Eq. (3). The shape matters for the substitution, because a car tour that is given up at a certain length can only become a walk or bike tour if walking or cycling is a relevant alternative at that length.

Response to a shock. Let V_a be the utility of alternative a and V_0 that of the no-tour option. In the nested logit, the inclusive value of nest n is $I_n = \ln \sum_{a \in n} e^{V_a}$, and the choice probabilities are

$$P(a \mid n) = \frac{e^{V_a}}{e^{I_n}}, \qquad P(n) = \frac{e^{\theta I_n}}{e^{V_0} + \sum_{n'} e^{\theta I_{n'}}}. \tag{4}$$

We take the base probabilities as observed, i.e. $P^0(n)$ and $P^0(a \mid n)$ are the shares of the tours T^0_{ca} together with a no-tour share $s_0 = 0.01$. The constants of the utilities are then implicit and need not be computed. A scenario adds Δ_a to V_a (Section 3.2), and the new probabilities are

$$P^1(a \mid n) = \frac{P^0(a \mid n)\, e^{\Delta_a}}{W_n}, \qquad P^1(n) = \frac{P^0(n)\, W_n^{\theta}}{P^0(0) + \sum_{n'} P^0(n')\, W_{n'}^{\theta}}, \qquad W_n = \sum_{a \in n} P^0(a \mid n)\, e^{\Delta_a}. \tag{5}$$

To derive Eq. (5), note that $e^{I^1_n} = \sum_{a \in n} e^{V_a + \Delta_a} = e^{I^0_n} \sum_{a \in n} P^0(a \mid n) e^{\Delta_a} = e^{I^0_n} W_n$. Inserting $e^{I^1_n}$ in Eq. (4) and dividing the numerator and the denominator of $P^1(n)$ by $e^{V_0} + \sum_{n'} e^{\theta I^0_{n'}}$ gives the stated expressions. The new tours are $T^1_{ca} = (N_c + N_{c0})\, P^1(n(a))\, P^1(a \mid n(a))$, where $N_{c0} = N_c\, s_0/(1 - s_0)$ is the no-tour mass of the cell.

Intuition. Eq. (5) lets the model compute any scenario directly from the observed tours, without estimating constants and without solving equations. The mechanism is simple. Within a nest, an alternative gains or loses in proportion to its own utility change relative to the average change in the nest, and the nest as a whole gains or loses in proportion to that average change raised to the power θ, so a θ below one dampens the movement between nests.

With $\theta = 1$, Eq. (5) reduces to the multinomial logit over all alternatives, and the ordering of the levels no longer matters. In structure B, the nests are the length nodes. A higher car cost then lowers the inclusive value of the long nodes most, so the tours shift toward shorter nodes with the weight θ, and within each node the car tours shift to other modes at the same length with weight one. In structure A, the nests are the modes, so a car tour that is given up first becomes a shorter car tour and only in the upper level, with weight θ, a tour by another mode. The kilometers of mode j are $K^1_{cj} = \sum_{a:j(a)=j} T^1_{ca}\, \ell_a$, and the tours and kilometers of a band are the sums over its nodes. Tours, kilometers and bands are therefore all output of the same choice model and add up by construction.

3.2 Generalized time, shocks and cost sharing

Generalized time. Level of service enters through generalized time. For a mode j with travel time T_j, money cost C_j and value of time v, generalized time is

$$G_j = T_j + \frac{s^{\mathrm{c}}_j\, C_j}{v}, \tag{6}$$

where $s^{\mathrm{c}}_j \in (0, 1]$ is the share of the money cost carried by the traveler. It is one for SOV and public transport and below one for the shared car modes. We define the money share of generalized time and the money-to-time odds as

$$\sigma_j = \frac{s^{\mathrm{c}}_j C_j / v}{G_j}, \qquad o_j = \frac{\sigma_j}{1-\sigma_j} = \frac{s^{\mathrm{c}}_j C_j}{v\, T_j}. \tag{7}$$

A scenario multiplies the money cost of mode j by $1 + x_{\mathrm{c}(j)}$ and its time by $1 + x_{\mathrm{t}(j)}$, where $\mathrm{c}(j)$ and $\mathrm{t}(j)$ are the cost and time shocks that apply to the mode. These are the car cost and car time for the three car modes, and the fare and public-transport time for public transport. Dividing the new generalized time by the old and using Eq. (7) gives

$$\frac{G'_j}{G_j} = 1 + \sigma_j x_{\mathrm{c}(j)} + (1-\sigma_j) x_{\mathrm{t}(j)}. \tag{8}$$

Eq. (8) depends on the levels of cost, time and value of time only through the money share σ_j. For this reason, the model can be calibrated and run without the cost and time of each origin and destination, which a surrogate of this kind does not have.

Utility change by tour length. In the nested logit, the change in the utility of alternative $a = (j, b, u)$ is

$$\Delta_a = -\beta_j \Big[\Big(\frac{\ell_a}{20}\Big)^{\varphi^{\mathrm{C}}} h^{\mathrm{C}}_j + \Big(\frac{\ell_a}{20}\Big)^{\varphi^{\mathrm{T}}} h^{\mathrm{T}}_j \Big], \qquad h^{\mathrm{C}}_j + h^{\mathrm{T}}_j = \ln \frac{G'_j}{G_j}, \tag{9}$$

for $j \in \mathcal{A}$, where h^{C}_j and h^{T}_j split the log change in generalized time into the parts due to cost and time in proportion to $\sigma_j x_{\mathrm{c}(j)}$ and $(1-\sigma_j) x_{\mathrm{t}(j)}$. Walk and bike have $\Delta_a = 0$ and are affected only through substitution. The exponents φ^{C} and φ^{T} are common to the three car modes, and public transport has its own pair. With $\varphi = 0$, the utility changes by $\beta_j \ln(G'_j/G_j)$ at every length. This is the logarithmic form of the cost-damping specifications of LTM (Rich and Mabit, 2016; Rich, 2020), and it leaves the length of the tours unchanged. With $\varphi = 1$, the change is roughly proportional to the length, as with a utility that is linear in generalized time. The calibrated exponents lie in between (Table 8), which corresponds to the damping that LTM obtains with its log-power spline. The exponent of the cost part is larger than that of the time part for the car. The cost per kilometer is constant with distance, whereas the time per kilometer falls on longer tours because the speed is higher, so a uniform cost increase has a larger effect on long tours than a uniform time increase. The tour length enters only through $\ell_a/20$, so β_j is the strength of the response at a tour of 20 km.

Cost sharing. Drivers with passengers and car passengers share the car cost. We use the cost shares of LTM 2.3, which follow Rich and Fox (2024) with the load factors of Rich (2022, Table 5.8). For each purpose, let L_{H} be the average number of adult passengers in a car with passengers, and L_{C} the average number of other adult occupants seen from a passenger. With equal sharing among adults and no sharing by children, the cost shares per person are

$$s^{\mathrm{c}}_{\mathrm{H}} = \frac{1}{1 + L_{\mathrm{H}}}, \qquad s^{\mathrm{c}}_{\mathrm{C}} = \frac{1 - 1/(1 + L_{\mathrm{C}})}{\max(1, L_{\mathrm{C}})}. \tag{10}$$

The driver with passengers pays one share out of $1 + L_{\mathrm{H}}$. The passenger's share is the part of the cost not paid by the driver, divided by the number of passengers, where the maximum

ensures that a passenger who travels on only part of the tour is not assigned more than a full share. Table 4 lists the resulting shares. The cost-sharing slider of the explorer multiplies both shares by a common factor λ^{c}, capped at one, so that $\lambda^{\mathrm{c}} = 0$ gives free-riding passengers and $\lambda^{\mathrm{c}} = 1$ gives the LTM shares. If a tour as driver with passengers and a tour as car passenger have the same cost and time per kilometer as a tour as driver alone, then C_j/T_j in Eq. (7) is the same for the three car modes, and the odds of the shared modes are

$$o_{\mathrm{H}} = s_{\mathrm{H}}^{\mathrm{c}}\, o_{\mathrm{S}}, \qquad o_{\mathrm{C}} = s_{\mathrm{C}}^{\mathrm{c}}\, o_{\mathrm{S}}, \qquad \sigma_m = \frac{o_m}{1 + o_m}. \tag{11}$$

Eq. (11) lets one free money share, σ_{S}, determine the money shares of all three car modes. Cost sharing is thereby a fixed part of the model rather than something the calibration can absorb. A passenger who pays half the cost experiences half the cost change for the same time change, and Eq. (11) expresses this in terms of the odds.

Table 4: Cost shares per person by purpose. Source: Rich (2022, Table 5.8) and Eq. (10).

Purpose	s_{HOV}	s_{CP}
Commute	0.525	0.459
Education	0.489	0.467
Escort	0.587	0.453
Shopping	0.493	0.452
Leisure	0.471	0.421
Business	0.468	0.396

3.3 Alternative models

Multinomial logit. The multinomial logit (MNL) is the nested logit with $\theta = 1$. It uses the same alternatives, length nodes, utility change Eq. (9) and calibration, and it differs only in the absence of the upper level. Every alternative then competes with every other in the same way within a cell. The aggregate responses still differ between modes, because the cells and the tour lengths differ. A higher car cost reduces the long car tours most, and the modes that are common at those lengths, mainly public transport, gain most. The MNL is the easiest model to explain, and Section 4.2 shows that it gives national car-kilometer results close to the nested logit. It gives a smaller shift to public transport.

IPDL over modes. The first version of the model used the inverse product differentiation logit of Fosgerau et al. (2022) over the six modes, with four overlapping nesting dimensions $\mathcal{G}_1, \ldots, \mathcal{G}_4$. The dimensions group active, car and public modes, all motorized modes, the two shared car modes, and walk alone against the other modes. The utility of mode j relative to the no-tour option is

$$\delta_j = \Big(1 - \sum_{d=1}^{4} \mu_d\Big) \ln s_j + \sum_{d=1}^{4} \mu_d \ln s_{g_d(j)} - \ln s_0, \qquad \Delta\delta_j = \beta_j \ln \frac{G_j'}{G_j}, \tag{12}$$

where $s_{g_d(j)}$ is the share of the group of j in dimension d, $\mu_d \geq 0$ and $\sum_d \mu_d < 1$. With one dimension, Eq. (12) is the nested logit with logsum parameter $1 - \mu_1$ (Berry, 1994). The new shares are found by Newton's method on Eq. (12), with a step that is accepted only if it stays inside the simplex and lowers the residual. The IPDL has no length dimension. The average tour length follows a log-linear pivot on the LTM length elasticities,

$$\Lambda_j(x) = \prod_{\kappa} (1 + x_\kappa)^{\eta_{j\kappa}}, \qquad \eta_{j\kappa} = \frac{\ln(1 + 0.1\, E_{j\kappa}^{\ell})}{\ln 1.1}, \tag{13}$$

and the response of each mode is split over the bands in proportion to the LTM band elasticities and rescaled to the total tours of the mode. The IPDL has two drawbacks. Firstly, its nests between modes have no counterpart in LTM, where substitution between modes arises within the mode and destination structure. Secondly, the length pivot and the band split are added outside the choice model, so the kilometers of the bands do not add up to the kilometers of the mode, and the band responses to combined shocks are not consistent with the responses to single shocks. The nested logit over mode and tour length has neither drawback.

4 Calibration and rebasing

4.1 Calibration on cells

Parameters. The parameters are calibrated per purpose. The nested logit has 16 free parameters: the logsum parameter $\theta \in (0.05, 1]$, the four coefficients $\beta_j > 0$ for $j \in \mathcal{A}$, the time exponents φ^{T} for car and public transport and the differences $\varphi^{\mathrm{C}} - \varphi^{\mathrm{T}}$, five share tilts, and the money shares σ_{S} and σ_{P}. The parameters are transformed so that $\theta = 0.05 + 0.95\,\mathrm{logit}^{-1}(t)$ and $\beta_j = e^{b_j}$, which keeps them in their ranges. The MNL has the same parameters without θ. The tilt shifts the shares of every calibration cell by mode,

$$T^0_{ca}(\alpha) \propto T^0_{ca}\, e^{\alpha_{j(a)}}, \qquad \sum_a T^0_{ca}(\alpha) = N_c, \qquad \alpha_{\mathrm{S}} = 0, \tag{14}$$

and absorbs the difference between the TU tour shares and the shares behind the LTM elasticities. The tilt is common to all cells of the purpose, so it does not change the differences between cells, and it is used only in the calibration.

Calibration cells. Each cell $c \in \mathcal{C}^{10}$ of the period 2009–11 has its own tours by mode, band and node, and its money-to-time odds are scaled by its value of time relative to the average traveler,

$$o_{c\mathrm{S}} = o_{\mathrm{S}}\,\phi_c, \qquad o_{c\mathrm{P}} = o_{\mathrm{P}}\,\phi_c, \qquad \phi_c = \frac{v(\bar{y})}{v(\bar{y}\,\rho_q)}, \tag{15}$$

where the shared car modes follow from Eq. (11). For shock κ at $x_\kappa = 0.1$, Eq. (5) gives the tours $T^{1(\kappa)}_{ca}$ of every alternative. The aggregate arc elasticity of the tours of mode j is

$$\hat{E}_{j\kappa} = \frac{1}{0.1}\left(\frac{\sum_{c\in\mathcal{C}^{10}}\sum_{a:j(a)=j} T^{1(\kappa)}_{ca}}{\sum_{c\in\mathcal{C}^{10}}\sum_{a:j(a)=j} T^0_{ca}} - 1\right), \tag{16}$$

and the mileage, length and band elasticities $\hat{E}^{\mathrm{km}}_{j\kappa}$, $\hat{E}^{\ell}_{j\kappa}$ and $\hat{e}_{jb\kappa}$ are computed in the same way from the kilometers, the kilometers per tour and the tours of each band.

Objective. The calibration minimizes

$$F = \sum_{j,\kappa}\Big[(\hat{E}_{j\kappa} - E_{j\kappa})^2 + w^2_{\mathrm{K}}(\hat{E}^{\mathrm{km}}_{j\kappa} - E^{\mathrm{km}}_{j\kappa})^2 + w^2_{\mathrm{L}}(\hat{E}^{\ell}_{j\kappa} - E^{\ell}_{j\kappa})^2\Big] + w^2_{\mathrm{B}}\sum_{j,b,\kappa}(\hat{e}_{jb\kappa} - e_{jb\kappa})^2 + R, \tag{17}$$

where the first sum runs over the 24 combinations of mode and shock, the second over the 48 band elasticities of the purpose, and $w_{\mathrm{K}} = 1$, $w_{\mathrm{L}} = 0.3$ and $w_{\mathrm{B}} = 0.1$. The penalty $R = 0.05^2\big(\sum_{j\neq\mathrm{S}}\alpha_j^2 + \sum_m(\mathrm{logit}\,\sigma_m - \mathrm{logit}\,\pi_m)^2 + \sum(\varphi^{\mathrm{T}})^2\big)$ pulls the tilts toward zero, the money shares toward their priors π_m (Section 4.3) and the time exponents weakly toward the logarithmic form. The mileage targets carry the same weight as the tour targets, because car vehicle-kilometers are the main output of the model. With a lower weight, the calibration fitted the tours slightly better but understated the car mileage elasticity of commuting, education and leisure by about 12%. The band elasticities carry a low weight, because they are noisy in the long bands with few tours. The no-tour share is fixed at $s_0 = 0.01$, which keeps the total number of tours almost constant, as in the LTM targets.

Algorithm. Eq. (17) is a nonlinear least-squares problem, which we solve by Levenberg–Marquardt. At each iteration, the Jacobian $\mathcal{J}_r$ of the residual vector r is computed by forward differences with step $h_i = 10^{-6}\max(1, |\vartheta_i|)$ for parameter ϑ_i, and the step $\Delta\vartheta$ solves

$$(\mathcal{J}_r^\top \mathcal{J}_r + \nu \operatorname{diag}(\mathcal{J}_r^\top \mathcal{J}_r))\,\Delta\vartheta = -\mathcal{J}_r^\top r, \tag{18}$$

with each component clipped to $[-3, 3]$. A step that lowers F is accepted and ν is divided by three. A step that does not is rejected, and ν is multiplied by four, up to twelve times per iteration. The iteration stops after 150 iterations, when no step lowers F, or when the relative improvement falls below 10^{-10}. The algorithm is run from three starting points, with $\beta_j \in \{e^{-1}, 1, e^{-0.5}\}$ and $\varphi \in \{0, 0.5, 1\}$, $\theta = 0.75$, zero tilts and the money shares at their priors. The best solution is kept. A full calibration of the six purposes takes about six seconds.

Aggregation. The aggregate elasticity in Eq. (16) is the tour-weighted average of the cell elasticities. The cell elasticity is a nonlinear function of the cell shares, and the weighted average of a nonlinear function differs from the function at the weighted average of its arguments. One representative cell with the average shares therefore does not reproduce Eq. (16), and the model is calibrated on the same cells that it is applied to. The weights favor the cells with many car tours, which are the two-car households with high car shares, and these cells respond weakly, because a mode with a high share has little room to lose. In the IPDL version, calibrated on the average traveler and then run on the cells, the commuting SOV elasticity to car cost fell from -0.153 to -0.095, i.e. by 38%, from the segmentation alone.

4.2 Fit to the LTM elasticities and choice of model

Fit. The elasticities of the model are output of the calibrated choice model and not the LTM numbers themselves, so every model elasticity differs somewhat from its target. Table 5 reports the root mean square error per purpose. For the nested logit, the error over the 24 tour elasticities lies between 0.017 and 0.029, and the error over the mileage elasticities between 0.017 and 0.035. The IPDL fits the tour elasticities more closely, with errors between 0.007 and 0.023, because it is calibrated to these targets alone and has nesting weights that act directly on the substitution between modes. The MNL fits worst. Table 6 shows every target and model elasticity for commuting, and Appendix B shows the same comparison for the other purposes. The largest differences for commuting are in the average length of walk and bike tours. LTM lets these tours become longer when car travel becomes more expensive, by 0.041 and 0.058 for car cost, whereas the model lets them become slightly shorter. In the model, the tours that move from car to walk or bike are short, because walking and cycling are only relevant at short lengths (Eq. (3)). In LTM, the same tours can move to destinations in other zones with a better walk or bike access, which a model without zones cannot represent. The walk and bike kilometers are small, so the effect on the totals is minor.

Bands. Table 7 compares the band elasticities of SOV tours to car cost and of public-transport tours to the fare. The model reproduces the increase of the response with the tour length. For commuting, the SOV tours below 20 km increase by 0.12 against 0.14 in LTM, because shorter car tours replace longer ones, and the SOV tours between 200 and 300 km decrease by 0.99 against 1.05. The largest differences are in the band above 300 km, where the model responds more strongly than LTM, and in the public-transport fare response of tours above 200 km for shopping and leisure, where it responds less strongly. These bands contain few tours, and LTM has a separate treatment of long-distance trips that the model does not represent.

Calibrated parameters. Table 8 lists the calibrated parameters. The money share of the car lies between 0.20 and 0.50, close to the prior of 0.37 for the private purposes. The length exponents of the time response lie between 0.1 and 0.5, and the cost exponent of the car is larger than the time exponent in every purpose, as discussed below Eq. (9). The calibrated logsum

Table 5: Calibration error per purpose: root mean square error (RMSE) of the nested logit over the 24 tour elasticities, the largest absolute error, and the RMSE over the length, mileage and band elasticities, compared with the RMSE of the MNL and IPDL over the tour elasticities, and the share of HOV among car drivers (TU 2009–11 and after the tilt). Source: model calibration on TU 2009–11 cells.

	NL					MNL	IPDL	HOV share, NL	
Purpose	Tours	Max	Length	Mileage	Bands	Tours	Tours	TU	Fit
Commute	0.019	0.045	0.037	0.023	0.165	0.042	0.007	0.084	0.086
Education	0.026	0.064	0.034	0.026	0.133	0.034	0.012	0.195	0.211
Escort	0.021	0.058	0.033	0.017	0.144	0.045	0.014	0.555	0.596
Shopping	0.029	0.068	0.030	0.030	0.204	0.051	0.023	0.335	0.224
Leisure	0.021	0.057	0.024	0.021	0.247	0.034	0.014	0.384	0.379
Business	0.017	0.039	0.033	0.035	0.107	0.055	0.013	0.107	0.095

Table 6: Commuting: LTM 2.3 arc elasticities at +10% and elasticities of the calibrated nested logit on the 2009–11 cells, for tours, average tour length and mileage. Diff. is model minus LTM. Source: Rich (2022, Table 6.28) and model calibration.

	Car cost			Car time			PT fare			PT time		
Mode	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.
Tours												
Walk	0.194	0.228	0.034	0.410	0.449	0.039	0.081	0.088	0.007	0.087	0.105	0.018
Bike	0.224	0.249	0.025	0.462	0.507	0.045	0.098	0.093	−0.005	0.112	0.113	0.001
SOV	−0.140	−0.155	−0.015	−0.206	−0.196	0.010	0.045	0.037	−0.008	0.062	0.049	−0.013
HOV	0.100	0.077	−0.023	−0.384	−0.393	−0.009	0.051	0.045	−0.006	0.070	0.061	−0.009
Passenger	0.093	0.072	−0.021	−0.386	−0.407	−0.021	0.090	0.075	−0.015	0.111	0.095	−0.016
PT	0.254	0.268	0.014	0.513	0.533	0.020	−0.528	−0.520	0.008	−0.667	−0.667	−0.000
Average length												
Walk	0.041	−0.026	−0.067	0.071	−0.043	−0.114	0.003	0.015	0.012	0.007	0.017	0.010
Bike	0.058	0.004	−0.054	0.090	0.014	−0.076	0.012	0.016	0.004	0.022	0.021	−0.001
SOV	−0.312	−0.284	0.028	−0.258	−0.267	−0.009	−0.003	0.003	0.006	−0.001	0.007	0.008
HOV	−0.202	−0.179	0.023	−0.385	−0.354	0.031	−0.003	0.009	0.012	−0.002	0.017	0.019
Passenger	−0.167	−0.130	0.037	−0.393	−0.370	0.023	0.008	0.007	−0.001	0.016	0.013	−0.003
PT	0.091	0.076	−0.015	0.130	0.104	−0.026	−0.178	−0.196	−0.018	−0.393	−0.400	−0.007
Mileage												
Walk	0.236	0.202	−0.034	0.484	0.404	−0.080	0.084	0.102	0.018	0.094	0.122	0.028
Bike	0.284	0.253	−0.031	0.556	0.522	−0.034	0.110	0.109	−0.001	0.134	0.133	−0.001
SOV	−0.448	−0.434	0.014	−0.459	−0.459	0.000	0.043	0.040	−0.003	0.060	0.057	−0.003
HOV	−0.104	−0.103	0.001	−0.754	−0.733	0.021	0.049	0.054	0.005	0.068	0.077	0.009
Passenger	−0.076	−0.059	0.017	−0.763	−0.762	0.001	0.099	0.082	−0.017	0.127	0.108	−0.019
PT	0.347	0.346	−0.001	0.649	0.642	−0.007	−0.697	−0.706	−0.009	−1.033	−1.041	−0.008

Table 7: Band elasticities at +10%: LTM 2.3 and calibrated nested logit on the 2009–11 cells, for SOV tours and car cost and for public-transport tours and fare. Bands in km, out and home. A dash marks a band without tours in the TU data. Source: Rich (2022, Tables 6.30–6.45) and model calibration.

		0–20	20–50	50–100	100–200	200–300	300+
SOV tours, car cost							
Commute	LTM	0.14	−0.02	−0.28	−0.67	−1.05	−1.06
	Model	0.12	−0.08	−0.33	−0.64	−0.99	−1.56
Education	LTM	0.13	−0.08	−0.40	−0.82	−1.04	−1.01
	Model	0.33	−0.16	−0.48	−0.78	−1.14	–
Escort	LTM	−0.04	−0.27	−0.54	−0.86	−1.08	−0.93
	Model	−0.08	−0.28	−0.46	−0.74	−0.92	−1.23
Shopping	LTM	−0.07	−0.30	−0.66	−1.01	−1.13	−1.06
	Model	−0.13	−0.40	−0.63	−0.88	−1.06	−1.30
Leisure	LTM	0.04	−0.24	−0.59	−1.01	−1.24	−0.85
	Model	−0.06	−0.29	−0.52	−0.81	−1.07	−1.62
Business	LTM	0.11	0.03	−0.08	−0.25	−0.42	−0.49
	Model	0.12	0.04	−0.07	−0.17	−0.28	−0.50
PT tours, fare							
Commute	LTM	−0.42	−0.46	−0.63	−0.87	−1.22	−1.08
	Model	−0.34	−0.46	−0.64	−0.80	−0.92	−1.07
Education	LTM	−0.19	−0.23	−0.36	−0.50	−0.62	−0.55
	Model	−0.14	−0.29	−0.40	−0.50	−0.59	−0.67
Escort	LTM	−0.76	−0.66	−0.70	−0.74	−1.03	−1.09
	Model	−0.73	−0.69	−0.68	–	–	–
Shopping	LTM	−0.70	−0.66	−0.82	−0.99	−1.35	−1.51
	Model	−0.65	−0.74	−0.90	−0.85	−0.93	−1.12
Leisure	LTM	−0.59	−0.51	−0.60	−0.80	−1.40	−1.40
	Model	−0.47	−0.65	−0.74	−0.91	−0.98	−1.10
Business	LTM	−0.30	−0.36	−0.50	−0.68	−0.88	−0.75
	Model	−0.28	−0.39	−0.54	−0.54	−0.75	−0.78

parameter lies between 0.15 and 0.76. It is below one in every purpose, so the model has less than perfect substitution across the upper level, as LTM has.

Table 8: Calibrated parameters of the nested logit per purpose: structure, logsum parameter θ, coefficients β_j, length exponents of the time and cost response for car and public transport, and money shares (prior and fitted). Source: model calibration on TU 2009–11 cells.

			β				φ, car		φ, PT		σ_{SOV}		σ_{PT}	
Purpose	Structure	θ	SOV	HOV	Pass.	PT	Time	Cost	Time	Cost	Prior	Fit	Prior	Fit
Commute	$p(m\|d)p(d)$	0.761	1.41	1.36	1.29	1.20	0.36	0.58	0.47	0.32	0.37	0.34	0.29	0.48
Education	$p(d\|m)p(m)$	0.154	3.84	3.62	3.29	4.89	0.10	0.26	0.16	0.06	0.37	0.39	0.33	0.47
Escort	$p(m\|d)p(d)$	0.528	1.75	1.69	1.78	1.61	0.15	0.48	0.34	-0.08	0.37	0.34	0.33	0.48
Shopping	$p(m\|d)p(d)$	0.588	1.64	1.50	1.47	1.71	0.30	0.39	0.31	0.11	0.37	0.50	0.35	0.54
Leisure	$p(m\|d)p(d)$	0.750	1.12	1.06	1.21	1.23	0.32	0.51	0.38	0.22	0.37	0.43	0.31	0.58
Business	$p(m\|d)p(d)$	0.603	1.17	1.21	1.23	1.21	0.33	0.57	0.42	0.29	0.12	0.20	0.08	0.39

The logsum parameter. The logsum parameter of the model is calibrated and is not the PLogsum of LTM, although the nesting structure is taken from LTM. The two parameters have the same role in the formulas, but they apply to different alternatives. In LTM, an upper-level alternative in structure B is a zone with its own size, its own level of service and its own mix of modes. In the model, it is a length node that aggregates all zones at the same tour length. A 20 km commute into central Copenhagen, with good public transport and expensive parking, and a 20 km commute in rural Jutland are the same node. The differences between such zones are not part of the model, and the calibration absorbs their effect on the elasticities in θ. When the zones in a node differ much, the node behaves less like one alternative, and the calibrated θ moves away from the LTM value. The coefficients β_j and the exponents φ also differ in meaning from the LTM coefficients, which act on a log-power spline of generalized time with zone-specific values. The calibrated θ is thus a reduced-form parameter that makes the aggregated structure reproduce the LTM elasticities. Fixing it at the LTM value would impose a parameter that belongs to a different set of alternatives. The structure is borrowed from LTM, whereas the value of θ is not.

Choice of structure. We also calibrated the nested logit with the LTM structure of every purpose and compared the two variants by cross-validation, described below. Table 9 reports the result. For commuting, education and business, the default structure is the LTM structure. For escort, shopping and leisure, LTM has the mode on top, structure A, and we use the length node on top, structure B, for three reasons. For escort, structure A predicts the left-out elasticities poorly, with an error of 1.24 against 0.27 for structure B. For shopping and escort, the calibration of structure A sets $\theta = 1$, so the nested logit collapses to the MNL, whereas structure B gives $\theta = 0.59$ for shopping, close to the LTM value of 0.567. For leisure, structure B predicts slightly better and gives $\theta = 0.75$ instead of 0.14 with structure A. A likely reason for the difference to LTM is the aggregation of zones into length nodes. When a node aggregates destinations with very different modes, a switch of mode at a given length is the closer substitute, which is what structure B expresses. For education, structure A is kept, although its calibrated θ of 0.15 is far below the LTM value of 0.504.

Cross-validation. A model with more free parameters fits better in sample, so the fit alone does not show which model represents the LTM substitution pattern best. We therefore compare the models by six-fold cross-validation. The 24 combinations of mode and shock of each purpose are divided into six folds of four. For each fold, the model is calibrated without the targets of these four combinations and then used to predict their tour elasticities. For the nested logit and the MNL, all targets of a held-out combination are removed together, i.e. the tour, length, mileage and band elasticities. This is necessary, because the mileage elasticity is the product of the tour and length elasticities, so a held-out tour elasticity would otherwise be implied by the targets that remain. Table 10 reports the root mean square error of the predictions, separately for the cross elasticities and for the own elasticities of the car modes to car shocks and of public

Table 9: Nesting structure per purpose: LTM 2.3 structure and PLogsum, and the calibrated logsum parameter θ and cross-validation RMSE of the nested logit with the LTM structure and with the default structure. $p(m|d)p(d)$ is structure B, $p(d|m)p(m)$ structure A. Source: Rich (2022, Tables 5.1 and 8.25) and model calibration.

	LTM 2.3		Model, LTM structure		Model, default structure		
Purpose	Structure	PLogsum	θ	CV RMSE	Structure	θ	CV RMSE
Commute	$p(m\|d)p(d)$	0.918	0.761	0.025	$p(m\|d)p(d)$	0.761	0.025
Education	$p(d\|m)p(m)$	0.504	0.154	0.043	$p(d\|m)p(m)$	0.154	0.043
Escort	$p(d\|m)p(m)$	0.506	1.000	1.239	$p(m\|d)p(d)$	0.528	0.269
Shopping	$p(d\|m)p(m)$	0.567	1.000	0.082	$p(m\|d)p(d)$	0.588	0.083
Leisure	$p(d\|m)p(m)$	0.560	0.137	0.063	$p(m\|d)p(d)$	0.750	0.057
Business	$p(m\|d)p(d)$	0.708	0.603	0.067	$p(m\|d)p(d)$	0.603	0.067

transport to public-transport shocks. The IPDL predicts the left-out cross elasticities best, with an error of 0.021 against 0.032 for the nested logit. It predicts the left-out own elasticities poorly, with an error of 0.441 against 0.207. Over all targets, the nested logit has the lowest prediction error, 0.122 against 0.255 for the IPDL and 0.508 for the MNL. The own elasticities are the hard case for every model. Each coefficient β_j is identified mainly by the own elasticity of its mode, and without that target it is determined by the cross elasticities and the penalties. The nested logit predicts them better than the IPDL, because the response of a mode to its own cost is tied to the same β_j that governs its response to the other shock of the same mode and its length and band responses. The pooled errors are dominated by escort, where the left-out elasticities are predicted with an error of 0.27 by the nested logit, 0.60 by the IPDL and 1.23 by the MNL. For the other five purposes, the errors of the nested logit lie between 0.025 and 0.083 and those of the IPDL between 0.019 and 0.117, so the two models predict about equally well outside escort. The MNL and the nested logit with the LTM structure, which has $\theta = 1$ for escort and shopping, predict the own elasticities poorly, with errors of 0.873 and 0.878. In the application, all targets are used, and the national results of the three models are close for car vehicle-kilometers. They differ more for public transport.

Table 10: Model alternatives: mean RMSE over the six purposes in sample (tours, mileage, bands), RMSE of left-out tour elasticities in six-fold cross-validation (cross, own, all), and national change from the 2023 base (%) in three scenarios. Source: model calibration and results.

	Fit (RMSE)			Cross-validation (RMSE)			Car cost +10%		Car time +10%	Fare −20%
Model	Tours	Mileage	Bands	Cross	Own	All	Car vkm	PT pkm	Car vkm	PT pkm
NL, mode × length (default structure)	0.022	0.026	0.167	0.032	0.207	0.122	−3.47	+2.51	−5.04	+16.98
NL, mode × length (LTM 2.3 structure)	0.032	0.036	0.158	0.055	0.878	0.509	−3.53	+2.29	−5.17	+16.98
MNL, mode × length	0.044	0.047	0.160	0.073	0.873	0.508	−3.44	+1.71	−5.29	+16.78
IPDL over modes, length pivot	0.014	–	–	0.021	0.441	0.255	−3.60	+2.47	−4.51	+16.00

4.3 Priors and rebasing to 2023

Priors. The priors π_m in Eq. (17) come from independent cost and time data. For the car, the money part per kilometer is $\gamma_{2010}/v^{\mathrm{T}}_{2010}$, where $\gamma_{2010} = 1.162$ DKK/km is the fuel and tyre cost and v^{T} is the TERESA value of time per minute (business or private), and the time part is 60/50 minutes per kilometer at 50 km/h. For public transport, the money part is $f(\bar{\ell}_{\mathrm{P}})/(\bar{\ell}_{\mathrm{P}}\, v^{\mathrm{T}}_{2010})$ at the mean public-transport tour length $\bar{\ell}_{\mathrm{P}}$ of the purpose, and the time part is 60/30 minutes per kilometer. The prior money share is the money part divided by the sum of the two parts. For the private purposes, the car prior is 0.37, and the calibration places the SOV money share between 0.34 and 0.50 (Table 8). This agreement suggests that the LTM cost definition and the TØE fuel and tyre cost are consistent. With the TERESA total operating cost, the prior was

0.64, and the calibration moved far away from it.

Rebasing. The calibrated θ, β, φ and cost shares are kept, and the model is moved to 2023 in three steps. Firstly, the tours of the cells by mode and band are replaced by the shrunk TU tours of the 2023 base, without the tilt. The GMM tables are not used as control totals, because they count trips and are about 3.2 times the TU tour totals. Secondly, the money-to-time odds are scaled by the growth of cost relative to the value of time,

$$r_{\text{car}} = \frac{\gamma_{2023}/\gamma_{2010}}{v_{2023}/v_{2010}} = \frac{0.936}{1.197} = 0.78, \tag{19}$$

where both the cost and the value of time are at fixed prices. For public transport, the corresponding ratio r_{PT} is still a placeholder and set to one. Thirdly, the odds of each cell are scaled by its relative value of time as in Eq. (15). The odds of a 2023 cell with income quintile q are therefore

$$o_{c\text{S}} = o_{\text{S}}\, r_{\text{car}}\, \frac{v(\bar{y})}{v(\bar{y}\,\rho_q)}, \qquad o_{c\text{P}} = o_{\text{P}}\, r_{\text{PT}}\, \frac{v(\bar{y})}{v(\bar{y}\,\rho_q)}, \qquad o_{cm} = s^{\text{c}}_m\, o_{c\text{S}},\ m \in \{\text{H}, \text{C}\}, \tag{20}$$

where o_{S} and o_{P} are the calibrated odds. Eq. (20) lowers the money share in high-income cells, where a given cost is worth fewer minutes.

Value of time. The value of time $v(y)$ is the LTM schedule of Rich (2022, Table 5.7), given as values $v_1, \ldots, v_{10}$ for personal income bands $[y^{\text{lo}}_n, y^{\text{hi}}_n)$ in 2010 DKK. We interpolate it linearly between the band midpoints $m_n = (y^{\text{lo}}_n + y^{\text{hi}}_n)/2$,

$$v(y) = v_n + (v_{n+1} - v_n)\frac{y - m_n}{m_{n+1} - m_n}, \qquad m_n \le y < m_{n+1}, \tag{21}$$

with constant values below the first and above the last midpoint. The open top band is closed at 1.5 times its lower bound, so its midpoint is at 1.25 times the lower bound, and the zero-income band is not used. The average personal income $\bar{y} = 252\,324$ DKK is that of the calibration period in 2010 prices, using a consumer price ratio of 1.27 between 2023 and 2010. The relative income ρ_q is the mean personal income of quintile q over the average. Table 11 shows the resulting values. The calibration in Eq. (15) uses the same relative value of time with $r = 1$, so that the income gradient is part of both the calibration and the application.

Table 11: Income (1000 DKK, 2023 prices), relative personal income ρ_q, value of time for commuting (2010 DKK per minute) and the 2023 odds scale of the car modes, $r_{\text{car}}\, v(\bar{y})/v(\bar{y}\rho_q)$ from Eq. (20). Source: TU and Rich (2022, Table 5.7).

Quintile	Household income	Personal income	ρ_q	VoT, commute	Odds scale, car
Q1	227	173	0.50	0.96	0.97
Q2	460	285	0.83	1.10	0.84
Q3	700	335	0.97	1.17	0.79
Q4	930	375	1.09	1.22	0.76
Q5	1 469	549	1.60	1.45	0.64

Effect of the rebasing. Table 12 compares the targets, the calibrated elasticities and the elasticities of the rebased model. For five of the six purposes, the rebasing reduces the SOV elasticity to car cost, mainly because the cost per kilometer has fallen relative to the value of time. For commuting, it changes from -0.155 in the calibrated model to -0.141 in 2023. For leisure, it rises from -0.202 to -0.221, because the change in the leisure travel pattern between the two periods outweighs the lower money share. The same mechanism raises the response to car time, because a lower money share means a higher weight of time in Eq. (8).

Table 12: Tour elasticities at +10%: LTM 2.3 target, calibrated nested logit on 2009–11 cells and rebased model on 2023 cells. Source: Rich (2022) and model results.

	SOV, car cost			HOV, car cost			PT, fare			PT, time		
Purpose	Target	2009–11	2023	Target	2009–11	2023	Target	2009–11	2023	Target	2009–11	2023
Commute	−0.140	−0.155	−0.141	0.100	0.077	0.092	−0.528	−0.520	−0.528	−0.667	−0.667	−0.677
Education	−0.202	−0.199	−0.176	−0.042	−0.092	−0.079	−0.258	−0.259	−0.262	−0.280	−0.296	−0.303
Escort	−0.191	−0.151	−0.124	−0.020	0.005	0.014	−0.723	−0.723	−0.531	−0.561	−0.572	−0.550
Shopping	−0.177	−0.224	−0.187	−0.052	−0.018	0.011	−0.709	−0.691	−0.733	−0.482	−0.522	−0.595
Leisure	−0.165	−0.202	−0.221	−0.009	−0.043	−0.071	−0.594	−0.586	−0.641	−0.410	−0.424	−0.482
Business	−0.070	−0.082	−0.053	0.126	0.127	0.111	−0.454	−0.442	−0.509	−0.826	−0.829	−0.989

5 Outputs and results

Tours, kilometers and occupancy. For a scenario $x = (x_{\mathrm{CC}}, x_{\mathrm{CT}}, x_{\mathrm{PC}}, x_{\mathrm{PT}})$, Eq. (5) gives the tours T^1_{ca} of every alternative in every 2023 cell, and the tours, kilometers and bands of a mode are their sums. Car vehicle-kilometers are the kilometers of SOV and HOV drivers, and occupancy is car person-kilometers (SOV, HOV and passengers) divided by vehicle-kilometers. Table 13 shows the national results for five scenarios for the three models together with a log-linear pivot on the LTM targets, $T^1_j/T^0_j = \prod_\kappa (1+x_\kappa)^{\tilde{e}_{j\kappa}}$ with $\tilde{e}_{j\kappa}$ converted as in Eq. (13). For a 10% increase in the car cost, the nested logit reduces car vehicle-kilometers by 3.5%, the MNL by 3.4% and the IPDL by 3.6%. Public-transport person-kilometers increase by 2.5%, 1.7% and 2.5%, respectively. The MNL gives a smaller shift to public transport, because without the upper level a car tour that is given up is spread over all alternatives in proportion to their shares, including the many walk and bike tours at short lengths. For a 10% increase in car time, the nested logit gives a larger reduction than the IPDL and the pivot. On the 2009–11 cells, its car mileage elasticity to car time is −0.423 against −0.434 in LTM. In 2023, the response is larger for two reasons. Firstly, the 2023 tours are distributed differently over lengths, and the response of the nested logit depends on the length through Eq. (9). Secondly, the lower money share after the rebasing gives time a higher weight in Eq. (8).

Table 13: National results for five scenarios, change from the 2023 base in percent, for the nested logit (NL), the multinomial logit (MNL), the IPDL and a log-linear pivot on the LTM targets. Occupancy in persons aged 10–84 per car. Source: model results.

	Car vehicle-km				PT person-km				All tours	Occupancy
Scenario	NL	MNL	IPDL	Pivot	NL	MNL	IPDL	Pivot	NL	NL (base 1.144)
Car cost +10%	−3.47	−3.44	−3.60	−3.76	+2.51	+1.71	+2.47	+2.56	−0.02	1.146
Car cost +46% (0.50 DKK/km)	−14.03	−13.98	−13.92	−14.04	+11.26	+7.58	+11.04	+10.65	−0.08	1.153
Car time +10%	−5.04	−5.29	−4.51	−4.24	+5.50	+4.10	+5.87	+5.14	−0.04	1.140
PT fare −20%	−0.92	−0.65	−0.61	−0.81	+16.98	+16.78	+16.00	+16.65	+0.01	1.143
PT time −10%	−0.63	−0.43	−0.36	−0.47	+12.47	+12.25	+11.16	+11.16	+0.00	1.144

Tour-length bands. In the nested logit and the MNL, the tours and kilometers of a band are the sums over its length nodes, so the bands add up to the totals of the mode in tours and in kilometers. A combined shock is handled by the same formula as a single shock, so the band responses to combined shocks are consistent with the responses to single shocks. The shocks are still uniform, and a charge that applies only to long tours cannot be represented, although the length nodes would allow such an extension.

Money burden. The money burden of cell c is computed by the rule of a half. For one weekday, the burden is

$$B_c = \sum_p \Big[\sum_{m\in\{\mathrm{S,H,C}\}} \gamma\, s^{\mathrm{c}}_m\, x_{\mathrm{CC}}\, \frac{K^0_{cm} + K^1_{cm}}{2} + (1-\delta_p)\, x_{\mathrm{PC}} \Big(26.2\, \frac{T^0_{c\mathrm{P}} + T^1_{c\mathrm{P}}}{2} + 0.92\, \frac{K^0_{c\mathrm{P}} + K^1_{c\mathrm{P}}}{2} \Big) \Big], \tag{22}$$

where $\gamma = 1.088$ DKK/km is the fuel and tyre cost (2025 prices), the fare consists of 26.2

DKK per tour and 0.92 DKK per kilometer, and δ_p is an optional season-pass discount that applies to commuting only. The per-kilometer parts use the average of the base and scenario kilometers, because a change in tour length changes the amount paid. The annual burden per person multiplies B_c by 300 weekday-equivalent days and divides by the number of persons aged 10–84 in the cell. Time losses are not valued, and the use of the revenue is outside the model. The burden is therefore the change in money outlay valued by the rule of a half, and not a complete welfare measure. Table 14 shows the burden and the change in car vehicle-kilometers by quintile and car ownership for a 10% increase in the car cost. The burden rises with the number of cars in the household. For persons in households with two or more cars, the annual burden is 867 DKK per person in the lowest quintile and 1 291 DKK in the highest. Relative to the mean personal income of the quintile, these amounts correspond to 0.50% and 0.24%, so the burden is about twice as large relative to income in the lowest quintile. The burden is in 2025 prices and the income in 2023 prices, so the shares are slightly overstated by the price growth between the two years.

Table 14: Car cost +10%, nested logit: annual money burden per person aged 10–84 (DKK, 2025 prices) and change in car vehicle-kilometers (%) by income quintile and household cars. Mean personal income in 1000 DKK, 2023 prices. Source: model results.

	Personal	No car		One car		Two or more	
Quintile	income	DKK	vkm %	DKK	vkm %	DKK	vkm %
Q1	173	50	−8.6	520	−4.1	867	−4.1
Q2	285	78	−6.7	660	−3.7	1 138	−3.0
Q3	335	68	−6.7	743	−4.5	1 311	−3.2
Q4	375	75	−5.9	697	−4.7	1 324	−3.0
Q5	549	64	−4.1	525	−4.0	1 291	−2.6

Validation. The implementation is checked by a test suite of 27 properties. For the nested logit, the suite verifies that the tour and mileage errors are below 0.035 and 0.04 per purpose, that θ lies in $(0, 1]$, that a zero shock leaves every cell unchanged, and that the cell totals equal the TU totals by band. It further verifies that the bands add up to the totals in tours and kilometers, that the band response to two opposing shocks equals the sum of the responses to each, that long SOV tours respond more than short ones, that the results are finite at every position of the sliders, that the own elasticities are negative after the rebasing, and that the burden is zero for a zero shock and positive for a cost increase. For the IPDL, the suite contains the corresponding checks and a check that the share solver converges at large reductions in travel time. The TU tabulation program is tested on synthetic data with the TU schema, and the workbook read in the browser gives the same calibration as the test path.

5.1 Experimental car-ownership costs

The explorer includes an experimental annual car-ownership cost slider for the NL and MNL models. It changes ownership upstream of transport demand. The sensitivity of ownership to the fixed cost is not estimated. It is set by two assumptions, which the user can change and which we intend to qualify later. Let q denote an income quintile, $k \in \{0, 1, 2\}$ the household car count, with two or more represented by two, and K the annual fixed cost per car. Household income Y_q and K are measured in 2023 DKK, and $\bar{Y}$ is the person-weighted mean household income. Utility is $A_{qk} - \gamma k K (Y_q/\bar{Y})^{-\lambda}$, with constants A_{qk} reproducing the observed person-weighted ownership shares separately in each quintile. For a scenario cost K', the resulting probabilities are

$$P'_{qk} = \frac{P^0_{qk} \exp\left[-\gamma k (K' - K)(Y_q/\bar{Y})^{-\lambda}\right]}{\sum_{h=0}^{2} P^0_{qh} \exp\left[-\gamma h (K' - K)(Y_q/\bar{Y})^{-\lambda}\right]}. \tag{23}$$

Eq. (23) shifts the baseline shares of each quintile by the change in annual cost, weighted by the number of cars and by the relative income of the quintile. The first assumption is the aggregate elasticity of $\sum_q N_q \sum_k kP_{qk}$ with respect to K, where N_q is the number of persons in quintile q. We calibrate γ to this elasticity, and the default is -0.4 following de Jong et al. (2009). The second assumption is λ, which determines how the cost sensitivity falls with income. The default $\lambda = 0.37$ is the slope of the LTM value of time with respect to personal income (Rich, 2022, Table 5.7). This slope corresponds to a money coefficient proportional to $Y^{-\lambda}$ at a constant time coefficient, and its transfer from trip costs to annual fixed costs is an assumption. With these defaults, the local elasticity falls from -1.07 in the lowest quintile to -0.20 in the highest. The aggregate elasticity does not depend on the reference cost K, because γ is recalibrated when K changes. The reference cost therefore only converts a change in DKK into the percentage of the slider, and it defaults to an editable, illustrative 20 000 DKK per car per year.

An earlier version used the utility $A_{qk} + \beta \log(Y_q - kK)$, with β calibrated to the cross-sectional income elasticity of 0.46. In that form, the probabilities depend on Y_q and K only through their ratio. Consequently, the cost elasticity equals minus the income elasticity, and a comparison with the fixed-cost elasticity of de Jong et al. (2009) is not an independent check. The earlier form also implied a local cost elasticity of -2.0 in the lowest quintile. The present form separates the two elasticities, and the earlier version corresponds approximately to $\lambda = 1$.

Connection to transport demand. The conditional weighting in Eq. (3) of Rich and Hansen (2016) provides the basis for connecting ownership to travel. For each original cell, the extension evaluates all three ownership states while holding the cell's other characteristics fixed. Differences in the primary ownership coefficients for SOV, HOV and passengers from Table 8.24 of Rich (2022) shift mode utilities. The frequency component uses the positive-frequency constants and ownership and logsum coefficients in Table 8.28, with the baseline zero-tour probability fitted to observed tours per person. For each mode and length outcome, expected conditional demand under the new ownership probabilities is divided by its counterpart under the original probabilities. This ratio multiplies the existing transport scenario outcome, preserving that outcome exactly when ownership costs are unchanged. The calculation is an aggregate approximation to conditional weighting, rather than a household transition model or a full replication of LTM. Distribution rows continue to describe baseline population groups, while a separate table displays the new marginal ownership shares.

Interpretation and validation. Higher fixed costs reduce ownership in this model and feed through to mode, frequency and mileage. With the default assumptions, a 10% increase reduces the capped person-weighted household car count by 4.02% and NL car vehicle-kilometers by 2.01% (2.36% for MNL). The ownership change follows from the assumed elasticity, so only the second figure is a model result. Each lost car reduces car vehicle-kilometers by about 10 km per person and weekday in the NL model. This reduction is between 46% and 67% of the cross-sectional difference between ownership groups in TU, depending on the quintile. The response is that of an average household in each cell. If the households that give up a car are those that drive least, the reduction in vehicle-kilometers is overstated. For comparison, de Jong et al. (2009) report a mileage elasticity of -0.16 with respect to fixed costs, rounded to -0.2 in their summary table. The mileage result is estimated among car-owning households and therefore does not identify the aggregate response mediated by ownership; dividing the two elasticities would not identify an ownership-to-use parameter. Controlled LTM runs separating changes from zero to one and from one to two cars are needed to assess the transferred coefficients and their utility scales. Transport costs and times do not feed back into ownership, and the extension is disabled for IPDL because its utility-scale mapping is not established. Fixed ownership costs are excluded from the monetary burden results, which therefore do not measure the total financial effect of this policy.

6 Discussion

Relation to standard models. A standard demand model estimates its parameters from observed choices and computes elasticities as an output. The present model inverts this order. It takes the elasticities of an estimated model as data and chooses the parameters of a simpler structure so that the structure reproduces them as closely as possible. This approach is sound as long as the structure can represent the substitution pattern of the original model, and the errors in Table 5 and Appendix B indicate that the nested logit over mode and tour length can. The approach has a cost in scope, because the model inherits the behavior of LTM 2.3 rather than the behavior in the data. The model will therefore not be more correct than LTM, and differences between LTM and GMM 4.0 (GMM 4.0 project, 2026), for example in the share of public transport, remain visible as differences between the targets and the 2023 travel pattern. The nested logit is preferred to the IPDL as the default, although the IPDL fits the tour elasticities more closely. Its structure is that of LTM, its bands and kilometers are consistent by construction, it predicts left-out elasticities better overall, and it needs no numerical solution of the share equations.

Positive own elasticities. The HOV driver has a positive elasticity to car cost for commuting and business, both in the LTM targets and in the model. A higher car cost makes some drivers who used to drive alone take passengers, and this inflow exceeds the loss from the higher cost carried by the HOV driver. The model reproduces this pattern, because cost sharing enters through Eq. (11) and an SOV tour that is given up moves partly to HOV at the same length.

Income. Income enters through two channels. The value of time in Eq. (20) lowers the money share and hence the car-cost response at high income for given shares. The tours of the cell by mode and length enter through Eq. (5). The two channels can work in opposite directions, and the model reports the composition effect as it appears in the data, without imposing a monotone income gradient. Removing the value-of-time gradient and recalibrating changes the national results very little, so the fit to the national LTM elasticities does not validate the income gradient itself. The distributional results are therefore conditional on the LTM value-of-time schedule and on the TU travel pattern of the cells.

Limitations. The model has six main limitations. Firstly, responses beyond ±20% follow the curvature of Eq. (9) and Eq. (5) and are not pinned by LTM results. LTM runs at −25%, +25% and +50% would provide additional targets, and the workbook reserves a sheet for them. Secondly, destinations are represented by tour length only, so differences between zones at the same length are absorbed by θ and the cells. This affects in particular the length of walk and bike tours and the long-distance bands. Thirdly, the calibration needs a large share tilt for car passengers in escort, which indicates that the TU and LTM shares of this mode differ. Fourthly, the experimental ownership-cost extension in Section 5.1 has not been validated against controlled changes in ownership. Its transfer of LTM coefficients to aggregate cells and the illustrative annual fixed cost require validation. Danish ownership dynamics and their dependence on income are analysed by Carrone and Rich (2021). Fifthly, the public-transport fare growth between 2010 and 2023 is a placeholder ($r_{\mathrm{PT}} = 1$), as are the 300 days used to annualize the burden, and the consumer price ratio of 1.27 between 2023 and 2010 is provisional. Finally, the primary-tour elasticities are applied to all tours, so the response of secondary tours is approximated, and the burden is reported per person aged 10–84 and not per household, because TU samples persons.

References

Dimitrios Argyros, Anders Fjendbo Jensen, and Jeppe Rich. Welfare-optimal public transport fares in Denmark. *Transportation Research Part A: Policy and Practice*, 204:104790, 2026.

Steven T. Berry. Estimating discrete-choice models of product differentiation. *The RAND Journal of Economics*, 25(2):242–262, 1994.

Andrea Papu Carrone and Jeppe Rich. A micro-model of car ownership dynamics: are preferences changing? *Transportation Letters*, 13(2):138–147, 2021. doi: 10.1080/19427867.2020.1716473.

Gerard de Jong, Marco Kouwenhoven, Karst Geurs, Pietro Bucci, and Jan Gerrit Tuinenga. The impact of fixed and variable costs on household car ownership. *Journal of Choice Modelling*, 2(2):173–199, 2009. doi: 10.1016/S1755-5345(13)70009-5.

DTU Management. TERESA 6.21, transportøkonomisk regnemodel, 2021. Spreadsheet, Technical University of Denmark.

DTU Management. Transportøkonomiske Enhedspriser, version 2.2, 2025a. Spreadsheet, Technical University of Denmark.

DTU Management. Transportvaneundersøgelsen (TU), data version 0624v1, 2025b. Danish National Travel Survey, documentation at tu2026.dk/meta.

Mogens Fosgerau, Julien Monardo, and André de Palma. The inverse product differentiation logit model. Working paper 3141041, SSRN, 2022.

GMM 4.0 project. Validering af elasticiteter i hverdagsdøgnmodellen (GMM 4.0). Technical Report OPD-BASISAAR-62-11 rev. F, draft, Grøn Mobilitetsmodel, 2026.

Thomas Ross Pedersen. Test of demand model for person trips in LTM, ver. 2.3. Note, Landstrafikmodellen, 2021. 5 March 2021.

Jeppe Rich. A spline function class suitable for demand models. *Econometrics and Statistics*, 14:24–37, 2020. doi: 10.1016/j.ecosta.2018.02.002.

Jeppe Rich. The weekday model – model for generation, destination and mode choice. Technical Report Doc. no. 35243-004 rev. 0, Landstrafikmodellen (LTM 2.3), 2022.

Jeppe Rich and James Fox. Cost sharing in passenger transport models: specification, implementation, and impacts. *Transportation Research Part A: Policy and Practice*, 179:103897, 2024.

Jeppe Rich and Christian Overgaard Hansen. The Danish National Passenger Model – model specification and results. *European Journal of Transport and Infrastructure Research*, 16(4), 2016. doi: 10.18757/ejtir.2016.16.4.3159.

Jeppe Rich and Stefan L. Mabit. Cost damping and functional form in transport models. *Transportation*, 43(5):889–912, 2016. doi: 10.1007/s11116-015-9628-8.

Jeppe Rich and Christian Anker Vandet. Is the value of travel time savings increasing? Analysis throughout a financial crisis. *Transportation Research Part A: Policy and Practice*, 124:145–168, 2019. doi: 10.1016/j.tra.2019.03.012.

A Input workbook and code

The input workbook `Data/elasticity_inputs.xlsx` contains one long table per sheet, and every row names its source. Table 15 lists the sheets that the model reads. The code is in the folder `code`. The Python programs in `code/data_prep` tabulate TU and build the workbook. The model is `code/engine/engine2.js`, its tests are in `code/engine/test_engine2.js`, and `code/engine/build_page.js` writes the explorer and a standalone file. The tables of this note are written by `code/engine/report_tables.js` and `code/engine/model_tables.js`, so that every number comes from the same code as the explorer.

Table 15: Sheets of the input workbook read by the model. Source: `code/data_prep/build_workbook.py`.

Sheet	Contents
`targets`	LTM 2.3 arc elasticities of tours, length and mileage (432 rows)
`band_elasticities`	LTM 2.3 tour elasticities by tour-length band
`vot_income`	LTM 2.3 value of time by personal income band and purpose
`load_factors`, `cost_shares`	Load factors and cost shares per person
`tu_journeys`	Tours and tour-kilometers by purpose, quintile, cars, mode and band, 2009–11 and 2023 (nested logit and MNL)
`cell_shares`	Shares, tours and lengths by purpose, quintile, cars and mode (IPDL)
`tu_persons`, `tu_income_quintiles`	Persons, car ownership and income by quintile
`teresa`, `toee_car_costs`	Value of time and car cost components
`costs`, `update_2023`	Fuel and tyre cost, fare, update ratios, with status
`curvature`	Reserved for LTM runs at larger shocks
`settings`	Model settings and placeholders

B LTM and model elasticities by purpose

Tables 16 to 20 compare the LTM 2.3 elasticities with those of the calibrated nested logit for the purposes other than commuting, which is shown in Table 6. Each table contains the tour, average-length and mileage elasticities for the four shocks at +10%, computed on the 2009–11 cells. Diff. is the model minus the LTM value.

Table 16: Education: LTM 2.3 and nested logit elasticities at +10%. Source: Rich (2022, Table 6.31) and model calibration.

	Car cost			Car time			PT fare			PT time		
Mode	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.
Tours												
Walk	0.022	0.039	0.017	0.062	0.095	0.033	0.070	0.088	0.018	0.065	0.099	0.034
Bike	0.036	0.040	0.004	0.089	0.098	0.009	0.095	0.088	−0.007	0.102	0.100	−0.002
SOV	−0.202	−0.199	0.003	−0.195	−0.234	−0.039	0.067	0.086	0.019	0.067	0.098	0.031
HOV	−0.042	−0.092	−0.050	−0.265	−0.296	−0.031	0.076	0.084	0.008	0.082	0.097	0.015
Passenger	−0.086	−0.048	0.038	−0.297	−0.233	0.064	0.105	0.085	−0.020	0.127	0.100	−0.027
PT	0.049	0.045	−0.004	0.110	0.105	−0.005	−0.258	−0.259	−0.001	−0.280	−0.296	−0.016
Average length												
Walk	0.008	−0.002	−0.010	0.016	−0.005	−0.021	0.014	0.002	−0.012	0.020	0.001	−0.019
Bike	0.014	−0.002	−0.016	0.026	−0.007	−0.033	0.019	0.003	−0.016	0.033	0.001	−0.032
SOV	−0.341	−0.305	0.036	−0.198	−0.173	0.025	−0.003	0.001	0.004	0.002	0.001	−0.001
HOV	−0.241	−0.225	0.016	−0.329	−0.256	0.073	0.001	−0.001	−0.002	0.008	−0.001	−0.009
Passenger	−0.178	−0.242	−0.064	−0.220	−0.321	−0.101	0.018	0.002	−0.016	0.032	0.001	−0.031
PT	0.011	−0.001	−0.012	0.018	−0.002	−0.020	−0.118	−0.113	0.005	−0.361	−0.325	0.036
Mileage												
Walk	0.029	0.037	0.008	0.078	0.090	0.012	0.083	0.091	0.008	0.085	0.101	0.016
Bike	0.050	0.038	−0.012	0.114	0.091	−0.023	0.114	0.090	−0.024	0.135	0.100	−0.035
SOV	−0.536	−0.498	0.038	−0.389	−0.403	−0.014	0.065	0.087	0.022	0.070	0.099	0.029
HOV	−0.282	−0.315	−0.033	−0.586	−0.545	0.041	0.078	0.083	0.005	0.090	0.096	0.006
Passenger	−0.262	−0.289	−0.027	−0.510	−0.546	−0.036	0.124	0.087	−0.037	0.159	0.101	−0.058
PT	0.060	0.044	−0.016	0.128	0.103	−0.025	−0.373	−0.369	0.004	−0.630	−0.612	0.018

Table 17: Escort: LTM 2.3 and nested logit elasticities at +10%. Source: Rich (2022, Table 6.34) and model calibration.

	Car cost			Car time			PT fare			PT time		
Mode	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.
Tours												
Walk	0.136	0.158	0.022	0.557	0.602	0.045	0.010	0.004	−0.006	0.006	0.002	−0.004
Bike	0.170	0.177	0.007	0.630	0.654	0.024	0.014	0.006	−0.008	0.010	0.004	−0.006
SOV	−0.191	−0.151	0.040	−0.095	−0.072	0.023	0.008	0.001	−0.007	0.007	0.001	−0.006
HOV	−0.020	0.005	0.025	−0.178	−0.173	0.005	0.008	0.001	−0.007	0.006	0.001	−0.005
Passenger	−0.006	−0.021	−0.015	−0.312	−0.342	−0.030	0.015	0.004	−0.011	0.012	0.004	−0.008
PT	0.233	0.233	0.000	0.759	0.701	−0.058	−0.723	−0.723	−0.000	−0.561	−0.572	−0.011
Average length												
Walk	0.038	0.005	−0.033	0.091	0.012	−0.079	0.001	0.001	0.000	0.002	0.001	−0.001
Bike	0.061	0.027	−0.034	0.115	0.086	−0.029	0.004	0.002	−0.002	0.005	0.001	−0.004
SOV	−0.253	−0.295	−0.042	−0.047	−0.094	−0.047	0.000	0.000	0.000	0.001	0.000	−0.001
HOV	−0.105	−0.119	−0.014	−0.055	−0.081	−0.026	0.000	0.001	0.001	0.001	0.001	−0.000
Passenger	−0.184	−0.153	0.031	−0.154	−0.105	0.049	0.002	−0.001	−0.003	0.003	−0.000	−0.003
PT	0.039	0.079	0.040	0.055	0.131	0.076	0.044	0.047	0.003	−0.173	−0.148	0.025
Mileage												
Walk	0.174	0.163	−0.011	0.653	0.615	−0.038	0.012	0.005	−0.007	0.008	0.003	−0.005
Bike	0.232	0.205	−0.027	0.752	0.745	−0.007	0.018	0.007	−0.011	0.015	0.005	−0.010
SOV	−0.439	−0.441	−0.002	−0.141	−0.165	−0.024	0.008	0.001	−0.007	0.007	0.001	−0.006
HOV	−0.125	−0.115	0.010	−0.232	−0.252	−0.020	0.008	0.002	−0.006	0.007	0.002	−0.005
Passenger	−0.190	−0.174	0.016	−0.460	−0.444	0.016	0.018	0.004	−0.014	0.015	0.003	−0.012
PT	0.273	0.313	0.040	0.818	0.842	0.024	−0.682	−0.680	0.002	−0.723	−0.711	0.012

Table 18: Shopping: LTM 2.3 and nested logit elasticities at +10%. Source: Rich (2022, Table 6.37) and model calibration.

	Car cost			Car time			PT fare			PT time		
Mode	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.
Tours												
Walk	0.164	0.181	0.017	0.228	0.264	0.036	0.052	0.047	−0.005	0.028	0.032	0.004
Bike	0.249	0.235	−0.014	0.332	0.344	0.012	0.063	0.051	−0.012	0.040	0.035	−0.005
SOV	−0.177	−0.224	−0.047	−0.112	−0.057	0.055	0.028	0.020	−0.008	0.022	0.017	−0.005
HOV	−0.052	−0.018	0.034	−0.212	−0.262	−0.050	0.031	0.021	−0.010	0.024	0.018	−0.006
Passenger	−0.088	−0.020	0.068	−0.277	−0.308	−0.031	0.044	0.036	−0.008	0.033	0.029	−0.004
PT	0.268	0.238	−0.030	0.376	0.343	−0.033	−0.709	−0.691	0.018	−0.482	−0.522	−0.040
Average length												
Walk	0.054	0.026	−0.028	0.065	0.035	−0.030	0.003	−0.001	−0.004	0.006	−0.001	−0.007
Bike	0.092	0.040	−0.052	0.108	0.054	−0.054	−0.001	0.012	0.013	0.007	0.008	0.001
SOV	−0.247	−0.277	−0.030	−0.155	−0.139	0.016	−0.003	0.005	0.008	0.000	0.006	0.006
HOV	−0.168	−0.179	−0.011	−0.299	−0.253	0.046	−0.005	−0.001	0.004	−0.001	0.001	0.002
Passenger	−0.166	−0.206	−0.040	−0.294	−0.292	0.002	−0.006	0.003	0.009	−0.001	0.005	0.006
PT	0.089	0.103	0.014	0.107	0.136	0.029	−0.044	−0.092	−0.048	−0.303	−0.225	0.078
Mileage												
Walk	0.219	0.207	−0.012	0.294	0.300	0.006	0.056	0.046	−0.010	0.035	0.032	−0.003
Bike	0.343	0.276	−0.067	0.443	0.400	−0.043	0.062	0.063	0.001	0.046	0.043	−0.003
SOV	−0.419	−0.494	−0.075	−0.266	−0.196	0.070	0.025	0.025	0.000	0.021	0.023	0.002
HOV	−0.220	−0.197	0.023	−0.505	−0.508	−0.003	0.026	0.021	−0.005	0.022	0.019	−0.003
Passenger	−0.252	−0.226	0.026	−0.562	−0.590	−0.028	0.038	0.039	0.001	0.032	0.035	0.003
PT	0.360	0.343	−0.017	0.487	0.483	−0.004	−0.750	−0.776	−0.026	−0.771	−0.734	0.037

Table 19: Leisure: LTM 2.3 and nested logit elasticities at +10%. Source: Rich (2022, Table 6.40) and model calibration.

	Car cost			Car time			PT fare			PT time		
Mode	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.
Tours												
Walk	0.136	0.164	0.028	0.293	0.350	0.057	0.065	0.048	−0.017	0.038	0.033	−0.005
Bike	0.176	0.166	−0.010	0.358	0.357	−0.001	0.074	0.061	−0.013	0.047	0.041	−0.006
SOV	−0.165	−0.202	−0.037	−0.087	−0.059	0.028	0.032	0.028	−0.004	0.024	0.022	−0.002
HOV	−0.009	−0.043	−0.034	−0.199	−0.230	−0.031	0.032	0.027	−0.005	0.024	0.022	−0.002
Passenger	−0.081	−0.053	0.028	−0.319	−0.336	−0.017	0.052	0.042	−0.010	0.039	0.031	−0.008
PT	0.176	0.174	−0.002	0.370	0.355	−0.015	−0.594	−0.586	0.008	−0.410	−0.424	−0.014
Average length												
Walk	0.035	0.003	−0.032	0.054	0.009	−0.045	−0.002	0.002	0.004	0.003	0.002	−0.001
Bike	0.060	0.027	−0.033	0.091	0.042	−0.049	−0.005	0.007	0.012	0.003	0.005	0.002
SOV	−0.347	−0.326	0.021	−0.201	−0.195	0.006	−0.003	0.007	0.010	−0.001	0.008	0.009
HOV	−0.250	−0.212	0.038	−0.357	−0.303	0.054	−0.004	0.007	0.011	−0.001	0.009	0.010
Passenger	−0.245	−0.216	0.029	−0.328	−0.329	−0.001	−0.006	0.011	0.017	−0.002	0.011	0.013
PT	0.064	0.073	0.009	0.093	0.093	−0.000	−0.208	−0.226	−0.018	−0.344	−0.326	0.018
Mileage												
Walk	0.171	0.167	−0.004	0.349	0.359	0.010	0.063	0.051	−0.012	0.041	0.035	−0.006
Bike	0.237	0.194	−0.043	0.452	0.400	−0.052	0.070	0.068	−0.002	0.049	0.046	−0.003
SOV	−0.506	−0.521	−0.015	−0.286	−0.253	0.033	0.029	0.035	0.006	0.023	0.030	0.007
HOV	−0.260	−0.255	0.005	−0.549	−0.526	0.023	0.028	0.035	0.007	0.023	0.032	0.009
Passenger	−0.324	−0.268	0.056	−0.637	−0.653	−0.016	0.045	0.053	0.008	0.037	0.042	0.005
PT	0.241	0.248	0.007	0.466	0.452	−0.014	−0.789	−0.799	−0.010	−0.740	−0.736	0.004

Table 20: Business: LTM 2.3 and nested logit elasticities at +10%. Source: Rich (2022, Table 6.43) and model calibration.

	Car cost			Car time			PT fare			PT time		
Mode	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.	LTM	Model	Diff.
Tours												
Walk	0.177	0.184	0.007	0.611	0.650	0.039	0.049	0.043	−0.006	0.051	0.065	0.014
Bike	0.180	0.165	−0.015	0.577	0.590	0.013	0.076	0.074	−0.002	0.087	0.108	0.021
SOV	−0.070	−0.082	−0.012	−0.087	−0.088	−0.001	0.023	0.023	0.000	0.049	0.047	−0.002
HOV	0.126	0.127	0.001	−0.254	−0.283	−0.029	0.028	0.022	−0.006	0.055	0.046	−0.009
Passenger	0.090	0.112	0.022	−0.363	−0.344	0.019	0.065	0.036	−0.029	0.098	0.064	−0.034
PT	0.232	0.238	0.006	0.866	0.886	0.020	−0.454	−0.442	0.012	−0.826	−0.829	−0.003
Average length												
Walk	0.029	0.011	−0.018	0.068	0.068	−0.000	0.004	−0.004	−0.008	0.023	−0.003	−0.026
Bike	0.061	−0.008	−0.069	0.104	−0.002	−0.106	0.008	0.019	0.011	0.018	0.026	0.008
SOV	−0.176	−0.186	−0.010	−0.300	−0.282	0.018	0.003	−0.000	−0.003	−0.027	0.005	0.032
HOV	−0.086	−0.074	0.012	−0.414	−0.371	0.043	0.007	0.005	−0.002	0.007	0.017	0.010
Passenger	−0.086	−0.059	0.027	−0.418	−0.469	−0.051	0.009	−0.005	−0.014	0.012	−0.002	−0.014
PT	0.083	0.082	−0.001	0.212	0.180	−0.032	−0.243	−0.261	−0.018	−0.774	−0.768	0.006
Mileage												
Walk	0.207	0.196	−0.011	0.685	0.722	0.037	0.053	0.039	−0.014	0.075	0.063	−0.012
Bike	0.242	0.157	−0.085	0.687	0.588	−0.099	0.084	0.094	0.010	0.105	0.135	0.030
SOV	−0.245	−0.266	−0.021	−0.385	−0.367	0.018	0.026	0.023	−0.003	0.022	0.052	0.030
HOV	0.039	0.052	0.013	−0.657	−0.644	0.013	0.035	0.027	−0.008	0.061	0.063	0.002
Passenger	0.003	0.052	0.049	−0.767	−0.797	−0.030	0.074	0.031	−0.043	0.110	0.062	−0.048
PT	0.317	0.322	0.005	1.097	1.082	−0.015	−0.686	−0.691	−0.005	−1.536	−1.534	0.002